Shubnikov–de Haas Oscillations in n-Type GaAs

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Shubnikov–de Haas oscillations in n-type GaAs are reported for four samples with carrier concentrations between 2×10^{16} and $2\times10^{17}/\text{cm}^3$. Temperatures employed ranged from 1.3 to 4.2°K, while magnetic fields were 34 kG and below. Scattering times were such that $0.1<\omega_c\tau<0.9$ for all measurements. Carrier concentrations derived from the oscillations agreed with Hall determinations to better than 10%. Only the fundamental oscillation in B^{-1} was observed, the phase angle being $\frac{1}{4}\pi$ within a considerable uncertainty forced by the extrapolation to $B^{-1}=0$. Amplitude variation with temperature and field for the two samples with carrier concentration greater than $10^{17}/\text{cm}^3$ was adequate to permit calculation of effective mass (0.058 ± 0.006) and Dingle temperature (~12°K). The magnitude of the amplitude agreed with theory to within a factor of 7.

I. INTRODUCTION

THE oscillatory magnetoresistance or Shubnikov–de Haas effect has been widely studied in compound semiconductors. For the III-V materials, Roth and Argyres[1] have reviewed the theoretical and experimental efforts prior to 1966. More recent work on these compounds has included further studies of GaSb,[2–5] InSb,[6–8] InAs,[7] and a single brief report of the oscillations in n-type GaAs.[9] We report here on independent observations of the oscillations in four n-type GaAs samples spanning the range $2\times10^{16}<n<2\times10^{17}$. For the two samples with $n>10^{17}$, envelope curves to the oscillations can be drawn with sufficient accuracy to yield effective-mass values and level-broadening information.

The theory of the Shubnikov–de Haas effect has been reviewed in Ref. 1. As we will show experimentally, harmonics of the magnetoresistance oscillations greater than the first can be neglected in the present case. In addition, the present doping levels and magnetic field range are such that the Fermi energy ζ is much greater than the cyclotron energy $\hbar\omega_c$ in which case the resistivity can be written

$$\rho\cong\rho_0[1+ab_1\cos(2\pi\zeta/\hbar\omega_c-\tfrac{1}{4}\pi)], \qquad (1)$$

where

$$b_1=-\left(\frac{\hbar\omega_c}{2\zeta}\right)^{1/2}\frac{\chi}{\sinh\chi}\cos(\pi\nu)\exp\left(\frac{-2\pi\Gamma}{\hbar\omega_c}\right),$$

$$a=1\,,\quad \text{for } I\parallel B$$
$$=\tfrac{5}{2}\,,\quad \text{for } I\perp B\,,$$

assuming an isotropic scattering mechanism,

$$\nu=gm^*/2m_e\,,$$

g being the gyromagnetic ratio for the electrons involved,

$$\chi=2\pi^2kT/\hbar\omega_c\,,$$

and

$$\Gamma=\text{Landau level width}=\pi kT'\,,$$

where T' is the Dingle temperature. Thus, in principle, by a study of these oscillations one can determine carrier density,[10] effective mass, level widths or scattering times, and g factor. In practice g cannot be evaluated since $gm^*/2m_e\sim0.013$ for GaAs[11] and the cosine function is therefore very insensitive to changes in g. Such measurements can also indicate anisotropy in the effective mass when present but none is expected for GaAs nor has any been detected here.

II. EXPERIMENTAL

Samples E-1 through E-3 were grown by vapor epitaxy methods on semi-insulating GaAs substrates. They were generally in the form of crosses with one leg broken off for layer thickness determination. Four Ohmic contacts were made on each sample, three near the ends of the existing legs and one at the point from which the fourth leg was removed. These contacts were formed by melting In into the surface in a 15% H_2–85% N_2 atmosphere in the presence of $AlCl_3$ flux. Hall and resistivity measurements were made on the three-legged arrangement and corrected for geometrical effects by comparing the room-temperature result with that determined prior to breaking the fourth leg. These corrections were typically 30%.

[1] L. M. Roth and P. N. Argyres, *Semiconductors and Semimetals*, edited by R. K. Willardson and A. C. Beer (Academic Press Inc., New York, 1966), Vol. I, p. 159.
[2] W. M. Becker and T. O. Yep, J. Phys. Soc. Japan Suppl. **21**, 366 (1966).
[3] D. G. Seiler and W. M. Becker, Phys. Letters **26A**, 96 (1967).
[4] A. I. Ponomarev and I. M. Tsidil'kovskii, Fiz. Tekh. Poluprovodnikov **1**, 1656 (1967) [English transl. Soviet Phys.—Semicond. **1**, 1375 (1968)].
[5] N. T. Sherwood and W. M. Becker, Bull. Am. Phys. Soc. **14**, 353 (1969).
[6] H. Miyazawa, H. Ikoma, and H. Maeda, Solid State Commun. **5**, 847 (1967).
[7] L. J. Neuringer and O. Beckman, Phys. Abstracts **71**, 1070 (1968), Abstract No. 13818.
[8] S. T. Pavlov, R. V. Parfenev, Yu. A. Firsov, and S. S. Shalyt, Zh. Eksperim. i Teor. Fiz. **48**, 1565 (1965) [English transl.: Soviet Phys.—JETP **21**, 1049 (1967)].
[9] A. V. Matveenko, G. D. Mel'nikov, and R. V. Parfen'ev, Fiz. Tverd. Tela **10**, 2842 (1968) [English transl. Soviet. Phys.—Solid State **10**, 2243 (1969)].
[10] For a parabolic band $2\pi\zeta/\hbar\omega_c=1.98\times10^{-6}n^{2/3}/B$.
[11] M. Cardona, J. Phys. Chem. Solids **24**, 1543 (1963).

Sample B-1 was a single-crystal bar cut from a polycrystalline boat-grown ingot. Contacts of In were fired at 350° as described previously.[12] In this case, Hall contacts were directly on the side of the bar so that no geometrical correction was necessary.

Magnetoresistance measurements were generally made using two contacts only but with separate current and voltage leads running to the contacts. Contact resistance, if present, would not affect the positions in magnetic field of the oscillations nor the field or temperature dependence of their amplitudes. Where absolute values of resistivity were of use (samples E-1 and B-1) separate four-point measurements were made.

Magnetic field was provided by a small superconducting solenoid with field homogeneity of better than 0.2% over the samples. This solenoid, made locally from Nb–48% Ti wire,[13] exhibited less than 10-G hysteresis for fields above 2 kG so that the magnet current provided a good measure of the field. The sample potential difference was largely balanced out by means of a potentiometer and the remaining signal amplified and recorded versus magnet current on an XY recorder. Both balancing voltage and sample current were stable to 0.01%. Sample orientations were accurate to approximately 5%.

III. RESULTS AND DISCUSSION

The oscillatory component of the magnetoresistance of the epitaxial samples is shown in Fig. 1. Only for sample E-1 were enough oscillations observed to permit construction of an envelope function which will be further analyzed below. The predicted amplitude ratio of $\frac{5}{2}$ between perpendicular and parallel field was not fulfilled. For example, at 2.65°K the ratio decreased with increasing field, going from 1.6 at 26 kG to 1.2 at 32 kG. Hall voltage oscillations of amplitude less than 0.04% were detected in this sample. This oscillatory component in $|V_H|$ was approximately 180° out of phase with the resistivity oscillations, i.e., minima of one corresponding in field to maxima of the other.

For samples E-2 and E-3 the data are insufficient to determine an envelope and thus the non-oscillatory magnetoresistance background curve from which the oscillations were measured must be considered no more than a reasonable extrapolation. This background curve exhibited negative magnetoresistance for all of the samples reported here. Such behavior has been studied recently in n-GaAs by Halbo and Sladek[14] and by Emel'yanenko *et al.*[15] For sample E-3 the subsequent positive-going $\Delta\rho/\rho$ was sufficiently strong above 25 kG

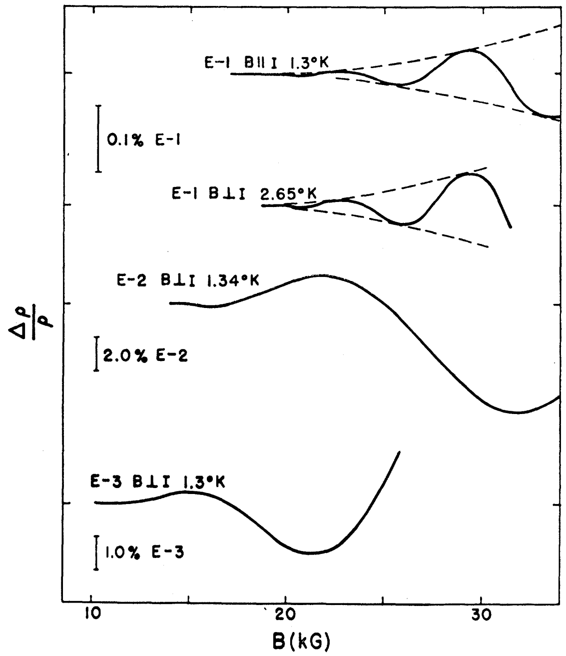

FIG. 1. Oscillatory part of the magnetoresistance for the epitaxial samples. Upward deflections indicate increasing resistivity.

to make the separation of an oscillatory part impossible. No further extrema were observed above this field in this sample.

Data for the bulk sample B-1 are shown in Fig. 2. Here variations in the Hall coefficient R were large

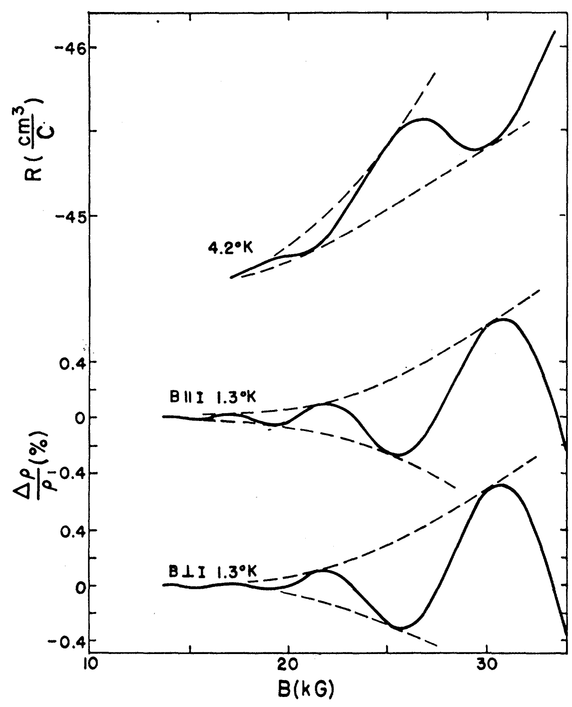

FIG. 2. Hall coefficient and resistivity oscillations for the boat-grown sample, B-1.

[12] A. H. Herzog, D. E. Hill, and J. W. Edwards, Semicond. Prod. **5**, 28 (1962).

[13] Norton Company T48B wire, 0.005-in.-diam core, 0.0065-in.-diam including Cu sheath.

[14] L. Halbo and R. J. Sladek, Phys. Rev. **173**, 794 (1968).

[15] O. V. Emel'yanenko, T. S. Lagunova, D. N. Nasledov, and G. N. Talalakin, Fiz. Tverd. Tela **7**, 1315 (1965) [English transl.: Soviet Phys.—Solid State **7**, 1063 (1965)].

TABLE I. Summary of results.

Sample	Dopant	n_S (10^{16}/cm^3)	n_H (10^{16}/cm^3) Low field	34 kG	μ (cm^2/V sec)	$\omega_c'\tau$	φ_0	T' (°K)	T'_m (°K)	m^*/m_e	Fig. 5 $B^{-1}=0$ Intercept
E-1	Sn	19	17.3	18.1	2400	0.49	0.7π	13.2	12.7	0.056 $\pm$0.008	0.15
B-1	Sn	12.5	14.1	13.6	1880	0.30	0.2π	11.6	16.2	0.058 $\pm$0.008	0.69
E-2	unknown	3.4	3.45	3.18	2010	0.27	0.3π		15.2		
E-3	unknown	2.0		2.04	1070	0.16	0.5π		28		

enough that the Hall voltage was recorded for both polarities of transverse field and differences read directly from the record. The result is shown in the top curve. It is clear from Fig. 2 that the oscillations in $|R|$ and those in $\Delta\rho/\rho$ are, to within experimental uncertainty, 180° out of phase. This was also true to more limited accuracy for sample E-1 above and appears to be the case in Ref. 4, although the sign of ΔR there is unspecified. In addition, the ratio of the transverse to longitudinal amplitudes is approximately 1.1 rather than the predicted 2.5. In this respect, however, the limited accuracy of our orientations (approximately $\pm5°$) should be kept in mind.

Generally accepted criteria for the observation of Shubnikov–de Haas oscillations are that $\hbar\omega_c > kT$, $\zeta > \hbar\omega_c$, and $\omega_c\tau \gg .1$[1,16] The first of these is very well obeyed for all of the present samples. The second would be violated for sample E-3 near the highest field available here but any oscillation appears to be masked by nonoscillatory magnetoresistance in that range. The third criterion ($\omega_c\tau \gg 1$) is definitely violated, as indicated in Table I. Here ω_c' represents the lowest cyclotron frequency at which definite oscillations were observed. The Hall mobility has been used in the mixed units of the table to calculate $\omega_c'\tau = 10^{-8}\,\mu B'$. In no case did the $10^{-8}\,\mu B$ product exceed unity in the present study. This was also true in the earlier GaAs report of Matveenko *et al.*[9]

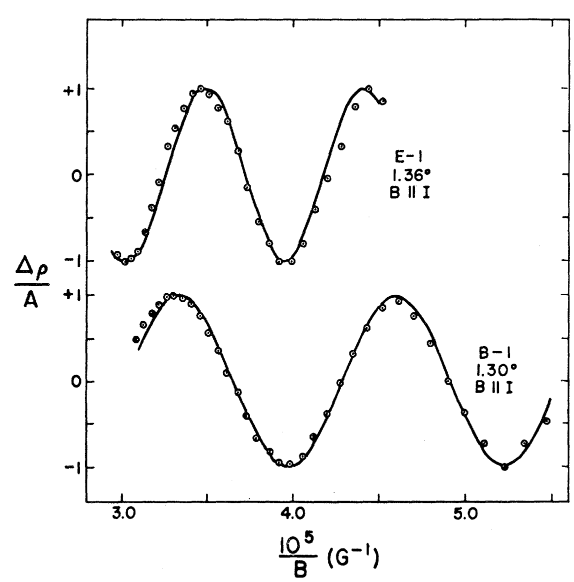

FIG. 3. The oscillatory voltage component normalized to its own amplitude envelope for samples E-1 and B-1.

Only the fundamental oscillation in B^{-1} is expected to be important under these $\omega_c\tau$ conditions. However, in order to be certain that Eq. (1) can be used without inclusion of higher harmonics the quantity $\Delta\rho/A$ has been calculated for the most interesting cases and is shown in Fig. 3. Here $\Delta\rho$ is the oscillatory component of the sample resistivity and A is the field-dependent amplitude of this quantity as determined from the envelope of the oscillatory component. If Eq. (1) is correct, $\Delta\rho/A$ should be a simple cosine function of $1/B$ and this is borne out in Fig. 3.

The period of the oscillations $\Delta(1/B)$ is related to the carrier density n_S through the Fermi energy in Eq. (1). The resulting equation is $n_S = [3.18\times10^6/\Delta(1/B)]^{3/2}$. In addition, one expects a phase difference φ_0 of $\frac{1}{4}\pi$ between the highest-field extremum predicted by Eq. (1) (a resistivity minimum) and the extrapolated point $B^{-1}=0$. In Fig. 4 the values of B^{-1} at which contact occurs between the oscillating component $\Delta\rho$ and the envelope function are plotted versus the implied argument of the cosine function in Eq. (1),[17] $\varphi = (2\pi/\hbar\omega_c) - \varphi_0 = l\pi$. The resulting values of n_S and φ_0 are shown in Table I along with carrier-density values obtained from Hall measurements in the liquid-helium range ($n_H = 1/|R|e$). The values of n are in agreement within 10% and, in addition, where a nonlinearity is discernible in Fig. 4 (for example, samples E-1 and E-2) it is in the direction indicated by the field-dependent Hall carrier density. Until the measurements are carried to higher fields permitting a smaller extrapolation to $B^{-1}=0$ the indicated values of φ_0 cannot be considered as seriously in disagreement with the theory.

To proceed further in the analysis reliable estimates of the resistivity amplitude A as a function of temperature and field are needed. At present these can be

[16] G. Landwehr, in *Physics of Solids in Intense Magnetic Fields*, edited by E. D. Haidemenakis (Plenum Press, Inc., New York, 1969), p. 415.

[17] Small deviations from periodicity in $1/B$ are expected for small l values [see H. P. R. Frederikse and W. R. Hosler, Phys. Rev. **108**, 1136 (1957) or Ref. 16]. This has been ignored in preparing Fig. 4.

extracted only for samples E-1 and B-1. A plot of

$$\log\left[\frac{A}{\rho}\left(\frac{2\zeta}{\hbar\omega_c}\right)^{1/2}\frac{\sinh\chi}{\chi}\right]\equiv\log\left(\frac{A}{\rho}F\right)$$

versus B^{-1} should yield a straight line of slope

$$-2\pi m^*c\Gamma/\hbar e$$

and intercept at $B^{-1}=0$ of log $\cos(\pi g m^*/2m_e)$. We have adopted the effective-mass value $m^*=0.07m_e$ in preparing Fig. 5. As in Fig. 3, points equally spaced in B have been read from the original continuous curves. Agreement with the theoretical field dependence is reasonable while order-of-magnitude agreement only is achieved with the expected intercept value $\cos(\pi\nu)=1.0$.[11] The values of T' and intercept arising from Fig. 5 are listed in Table I together with values of T'_m from the Hall mobility ($T'_m=e\hbar/2\pi m^*k\mu$). The effective mass enters Fig. 5 through the factor χ. As we will see below, a value of $0.058m_e$ is indicated by these experiments. Had such a value been used in preparing Fig. 5, the systematic differences with temperature would be largely eliminated. The slope and T' would not be changed significantly, however. Measurements taken in transverse field displayed a larger variation of slope and intercept in a plot similar to Fig. 5.

Effective-mass values are deduced from the data by forming the ratio $A(T_2,B)/A(T_1,B)$ for various

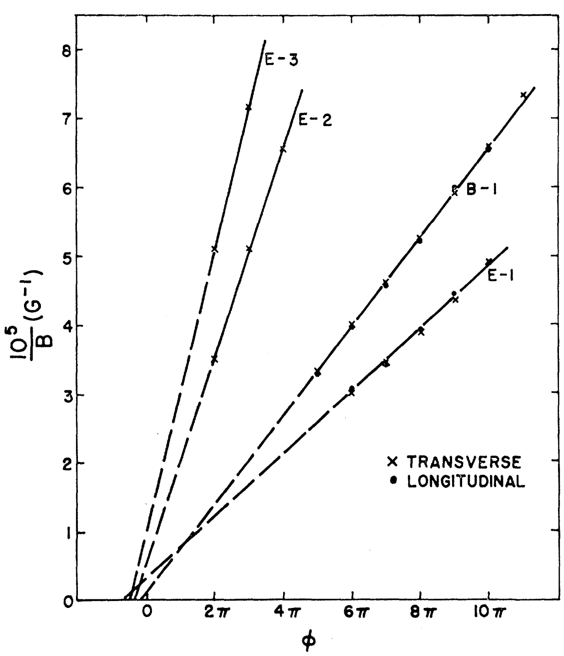

FIG. 4. $1/B$ values at which the oscillatory voltage component contacts its amplitude envelope plotted versus φ at consecutive integer values of l for $\varphi=l\pi$. The choice of l is made such as to cause the $B^{-1}=0$ intercept to be as close as possible to $-\frac{1}{4}\pi$ while still demanding l be odd for resistivity maxima.

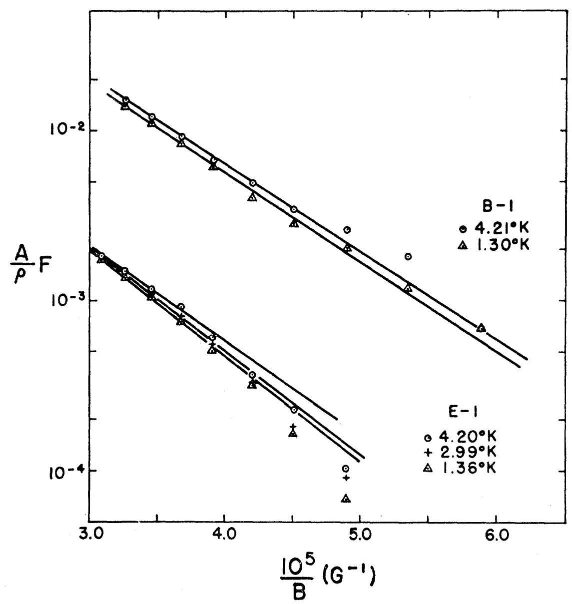

FIG. 5. Semilog plot of normalized oscillatory resistivity amplitude versus $1/B$.

values of the magnetic field. The theory indicates that this should equal $T_1 \sinh\chi(T_2,B)/T_2 \sinh\chi(T_1,B)$, a function only of B, T_1, T_2, and m^*. All values of effective mass determined from the present data are smaller than $0.07m_e$, both for longitudinal and transverse fields. Once again the longitudinal results are the more consistent and they alone will be considered below.

These data, involving five temperatures for each of the two specimens, were treated as follows: Data from two temperatures were combined to yield effective-mass values at intervals of 1.7 kG in the range of trustworthy amplitudes. Each m^* was assigned a weight given by the product of the mean amplitude and the temperature difference for that field and pair of temperatures. The m^* values (approximately 20 for each sample) resulting from all useful temperature pairs were combined into a weighted average and weighted mean absolute deviation. The results are $m^*=(0.0563\pm0.0041)m_e$ for sample E-1 and $m^*=(0.058\pm0.0039)m_e$ for sample B-1. In further combining these values into the single m^* quoted in the abstract we have continued to use the weighting factors, a procedure which favors the larger amplitude B-1 results. The uncertainties listed earlier and in the table are seen to be twice the mean deviation or that number reduced by $1/\sqrt{2}$ for the combined value. These figures reflect reasonable allowances for possible systematic errors but give no consideration to approximations in the theory.

The effective-mass values determined here are low, even compared to the recent determinations of

$m^* = 0.066m_e$.[18,19] This value, appropriate to the bottom of the band and zero temperature, would be expected to increase to approximately $0.068m_e$ for the present temperature and carrier concentrations.[18] Thus, a discrepancy exists which, however, may be associated with the small values of $\omega_c\tau$.[1,20] No systematic variation in m^* with B (and therefore $\omega_c\tau$) has been observed here, however. It is to be noted that there is general agreement between effective-mass values from Shubnikov–de Haas and other measurements for other materials.

[18] H. Pillar, J. Phys. Soc. Japan Suppl. **21**, 206 (1966).
[19] G. E. Stillman, C. M. Wolfe, and J. O. Dimmock, Solid State Commun. **7**, 921 (1969).
[20] E. N. Adams and T. D. Holstein, J. Phys. Chem. Solids **10**, 254 (1959).

IV. SUMMARY

The existence of Shubnikov–de Haas oscillations in the magnetoresistance of n-type GaAs has been demonstrated for the carrier-density range $2\times10^{16}/\text{cm}^3 < n < 2\times10^{17}/\text{cm}^3$. As with the more widely studied III-V semiconductors, reasonable agreement with theory is obtained for both the field and temperature dependence of the amplitude of the oscillations. From the latter an effective-mass value of $(0.058\pm0.006)m_e$ at $1-2\times10^{17}$ is obtained.

Note added in proof. S. Askenazy, J-P Ulmet, J. Léotin, L. Holan, and A. Laurent have recently reported Shubnikov-de Haas oscillations in an n-type GaAs sample with 4×10^{17} electrons/cm^3 [Solid State Commun. **7**, 717 (1969)].

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Far-Infrared Recombination Emission in n-Ge and p-InSb†

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Far-infrared recombination emission with wavelengths longer than 90 μ was observed under pulsed conditions from the impact ionization of impurity in n-Ge(Sb) and p-InSb samples at 4.2°K. A fast-response n-InSb free-electron bolometer was used as the detector and various filters were used to isolate the spectral regions. Emission with $h\nu$ near 4 meV is interpreted as recombination of free carriers with the lowest excited state of the impurity for both n-Ge and p-InSb. For the n-Ge sample, emission with $h\nu\sim7.8$ meV is attributed to electron transitions from an excited state to the ground state. There was also some indication of emission with photon energies larger than the ionization energy of 9.7 meV of Sb. For p-InSb, emission was observed at photon energies near the impurity ionization energy of about 7.5 meV. This is attributed to the recombination of free holes with the ground state of the impurity. Under certain conditions of pulse duration and electric field, an afterglow was observed after the electric field was removed from the samples. An explanation is suggested for this phenomenon.

I. INTRODUCTION

FAR-INFRARED radiation of about 100 μ from the recombination of impact-ionized donors in n-type Ge samples doped with Sb and As at 4.2°K was first reported by Koenig and Brown.[1] The signal was detected by observing the photoconductive response of a second sample of Ge which was doped with a donor different from the one used for the generator. The sample was partially broken down by applying a dc voltage. The amount of radiation detected was exceedingly small, about 1.5×10^{-12} W. The radiation was attributed to electrons with kinetic energy of about $10kT$ recombining with the donor ground state or to unreasonably energetic electrons recombining with an excited state of the donor. A second study concerning the recombination radiation of Sb donor in Ge was reported by Ascarelli and Brown.[2] A block of Sb-doped Ge was used as detector. The sample, Ge with smaller Sb doping, was inserted into a hole bored in the detector. A large increase of noise of the detector was observed when an applied electric field produced a breakdown of the sample. The effect was attributed to recombination radiation given by the sample.

The present work on far-infrared emission from impact-ionized n-Ge(Sb) and p-InSb was undertaken to obtain more understanding of the phenomena. The emission, together with the sample conductance, was studied as functions of the applied electric field. Various filters were used to obtain information on the spectrum of emission which facilitates the interpretation in terms of electron transitions. The strongest signals measured involve excited states and they could not have been observed with techniques which are not sensitive for these wavelength regions. The emission from p-InSb was measured for the first time. An afterglow emission

† Work supported in part by a contract from the Advanced Research Projects Agency.
[1] S. H. Koenig and R. D. Brown, III, Phys. Rev. Letters **4**, 170 (1960).
[2] G. Ascarelli and S. C. Brown, Phys. Rev. **120**, 1615 (1960).